ELASTO-VISCOPLASTIC MODELLING OF NON LINEAR BEHAVIOUR OF ASPHALT

K. Nesnas[1]
[1]*TRL limited, Old Wokingham Road, Crowthorne, Berkshire RG45 6AU*

ABSTRACT

Current design procedures assume that the response of pavement systems to wheel load or temperature variations is elastic. The domain of validity of this assumption is restricted only to low temperatures and high speeds where the asphalt layers of the pavement behaves in an elastic manner. At high temperatures the asphalt is subjected to plastic flow associated with viscous behaviour. Clearly the need for a new constitutive model to represent the mechanical response of the upper layers of the pavement is necessary. The main components of the model are: elasticity to define the instantaneous response of the model, viscosity to define the dependency of the mechanical response on the strain rate and hence on the frequency of loading, plasticity to define occurrence of the plastic flow and hence permanent deformation. All model parameters such the Young modulus, the yield stress, the viscosity and the hardening slope in the plastic range are made temperature dependent in order to reflect the sensitivity of asphalt to temperature variations. The paper gives a brief description of the constitutive model based on elasto-viscoplasticity with kinematic hardening and discusses the prediction of some laboratory tests.

Keywords: Permanent deformation, rate of loading, temperature, viscosity.

1. INTRODUCTION

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Constitutive modelling of asphalt material has lagged behind constitutive modelling of materials such as soils, concrete and metals. The main reason for this is that linear elastic theory has been considered adequate for pavement modelling. The assumption of linear elasticity is correct only at low temperature which produces a high modulus of the asphalt material, and for roads subjected to high-speed traffic.

With slow moving vehicles and hot weather conditions the pavement response is not elastic. As the viscous characteristics of the asphalt are more important and the deformations of the asphalt layers in the pavement are more likely to be controlled by the viscous and the plastic components.

Clearly, to correctly predict the movements in the asphalt layer and the stress distributions requires a constitutive model, which is able to characterise correctly the load spreading ability in pavements. Therefore the constitutive model should account for the following characteristics of the mechanical behaviour of the asphalt material:

- Temperature: At high temperatures the mechanical behaviour of the asphalt is very dependent on strain rate, due to its viscous nature; at low temperature the behaviour is brittle.

- Frequency of loading: The speed of load application and hence vehicle speed influences the material stiffness. This is very dependent on vehicle speed.

- Anisotropy: It is related to the nature of the material used in pavement construction. Two types of anisotropy can be distinguished: the spatial anisotropy and the mechanical anisotropy. The former relates to material property variations due to material non-homogeneity and the latter relates to change in material strength with plastic straining and the occurrence of different yield stresses for the tension and compression.

- Plastic flow: The occurrence of plastic flow results in stiffness reduction.

2. BRIEF DESCRIPTION OF THE ELASTOVISCOPLASTIC MODEL

The model assumes that viscoplastic strain will not occur until the stress σ is greater than the yield stress σ_y (defining the onset of plasticity). Thus the stress-strain relationship is written as:

$$\sigma = D(\varepsilon - \varepsilon^p) \tag{1}$$

Where σ is the applied stress, D is the tensor of elastic moduli based on Hooke's elasticity, ε is the total strain and ε^p is the viscoplastic strain.

2.1 ELASTICITY

The material elasticity is defined by the following stress-strain relationship relating stress to elastic stain as:

$$\sigma = D\varepsilon^e \tag{2}$$

The components of the tensor of elastic moduli D are defined as:

$$D_{ijkl} = \lambda\delta_{ij} + \mu\left[\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right] \tag{3}$$

Where λ is the Lame constant for volume and μ is the Lame constant for shear and δ is the Kronecker constant ($\delta_{ij}=0$ if i=j and $\delta_{ij}=1$ if $i \neq j$). λ and μ are made temperature dependent.

2.2 YIELD CRITERION

Kinematic hardening is assumed, it corresponds to a translation of the elastic domain in the stress space during viscoplastic flow. Therefore the stress state is defined by the stress σ and the position of the centre of the yield surface χ. In the following σ and χ are considered to be deviatoric as volumetric deformations in the plastic range are assumed to be negligible.

The onset of viscoplasticity is defined by a simplified version of the general equation of the yield surface [1]. The yield surface is defined as:

$$F = J_2(\sigma - \chi) - \sigma_y - k(\varepsilon_p) = 0 \tag{4}$$

Where $J_2=[2/3(\sigma-\chi):(\sigma-\chi)]^{1/2}$ is the second stress invariant for shear, σ_y is the temperature dependent yield stress, k is the hardening parameter and it is a function of the plastic strain ε^p given as:

$$\dot{k}(\varepsilon^P) = \dot{\chi} = \frac{2}{3}C\dot{\varepsilon}^p - \vartheta\chi\dot{\varepsilon}_p \tag{5}$$

Where the dot symbol indicates first derivative with time; ε_p is the magnitude of the plastic strain ε^p; C and ϑ are coefficients which are dependent on temperature.

Equation (5) was introduced by Armstrong and Frederick [2] for the time-independent plasticity. It is based on an evolution equation of the internal variable χ containing two terms: the first term expresses a linear kinematic hardening and the second term expresses a fading memory effect of the deformation path.

2.3 FLOW RULE

The flow rule relating the plastic strain rate to the current stress state is defined based on the Odqvist's law [3] which is a generalization of the well known Norton's law (expressed as a power law of the creep stain related to stress) by considering the elastic domain to be negligible. For this work, a version of the flow rule with an elastic domain developed by Lemaitre et al [4] is used:

$$\overset{\bullet}{\varepsilon}^{p} = \frac{3}{2}\left[\frac{\left(J_2(\sigma-\chi)-\sigma_y-k(\varepsilon^{p})\right)}{\eta}\right]^{m}\frac{\sigma-\chi}{J_2(\sigma-\chi)} \tag{6}$$

Where $\overset{\bullet}{\varepsilon}^{p}$ is the viscoplastic strain rate and η and m are constant temperature dependent coefficients.

3. PARAMETRIC IDENTIFICATION

Five parameters need to be identified in order to define the constitutive relations, these parameters are: The elastic parameters in terms of the Young Modulus E and the Poisson's ratio ν, the Yield stress σ_y, the viscous characteristics η and the parameter m, and the hardening modulus.

The elastic modulus E is obtained from the initial slope of the stress-strain curves. A value of 0.35 is assumed for Poisson's ratio, generally this value is used for asphalt material. The third parameter which is the yield stress is obtained directly from the relaxation curves. The viscous characteristic η and the parameter m are obtained from fitting Equation (6) to experimental data by using a non-linear least square algorithm [5].

The hardening modulus identifies with C in Equation (6). It is obtained from the relationship linking the variation of the viscoplastic strain with stress level. This relationship is obtained from a plot of stress as a function of viscoplastic strain. The coefficient ϑ is equal to zero.

4. THEORETICAL PREDICTION OF LABORATORY TESTS

In the following, the mechanical behaviour of asphalt under compression, relaxation and cyclic loading is predicted using the theoretical model. Experimental data [6] showed that the asphalt material is very sensitive to strain rate as observed when plotting the stress-strain curves for asphalt at different strain rates. Also they showed that the asphalt material is very viscous. The effect of strain rate and viscosity increases with an increase of temperature.

4.1 Compression

The prediction of the material response to compression are show in Figures 1 and 2 for a temperature of 20^{o}C. Figure 1 shows the effect of the rate of loading on the stress-strain curve for a temperature of 20^{o}C, and Figure 2 shows the effect of temperature on the asphalt response for a rate of loading of $\partial\varepsilon/\partial t=3\%s^{-1}$. The theoretical results are in good agreement with the experimental results in the pre-peak region. In the post peak region the theoretical model over predicts the experimental results which show a decrease in the stress. This reduction in the stress is because of the damage occurred in the specimen. The modelling of this aspect, which requires coupling of viscoplasticity with damage, has not been attempted yet in the current project. Overall the model predictions are excellent as the right stress level were predicted for all loading rates and temperatures. Most importantly the following observations are made:

- The mechanical response of the asphalt under compression is highly non-linear as it is very dependent on the rate of loading and the temperature of the specimens.
- Failure load increases as the loading rate increases and the temperature decreases.
- The initial slopes of the stress-strain curves are dependent on the rate of loading and the temperature.

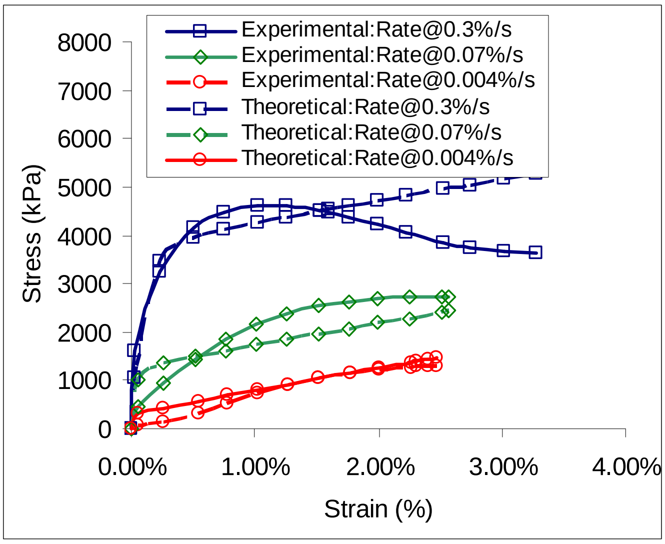

Figure 1: Theoretical predictions of compression at a temperature of 20^{o}C for different strain rates

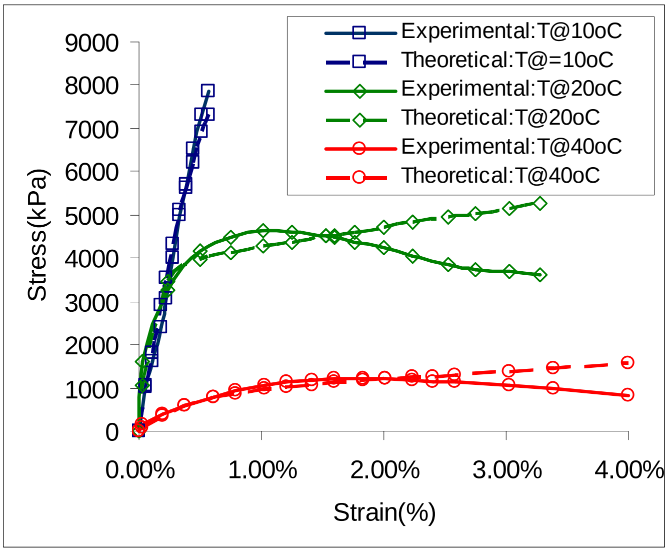

Figure 2: Theoretical predictions of compression at a strain rate of 0.3%/s for different temperatures

4.2 Relaxation

The response of asphalt to a compressive strain of 5% sustained for a period of 500 seconds is shown in Figure 3 for a temperature of 10^{o}C and in Figure 4 for a temperature of 20^{o}C. The dashed line represents the experimental curve and the solid line represents the theoretical curve. The results showed that the asphalt exhibited large stress reduction due to relaxation. All the curves reached an asymptotic value defining the yield stress. The yield stress reduces as the temperature increases. Overall the predictions of the theoretical model are excellent as the model parameters obtained from the calibration of the compression tests were used to predict the relaxation tests.

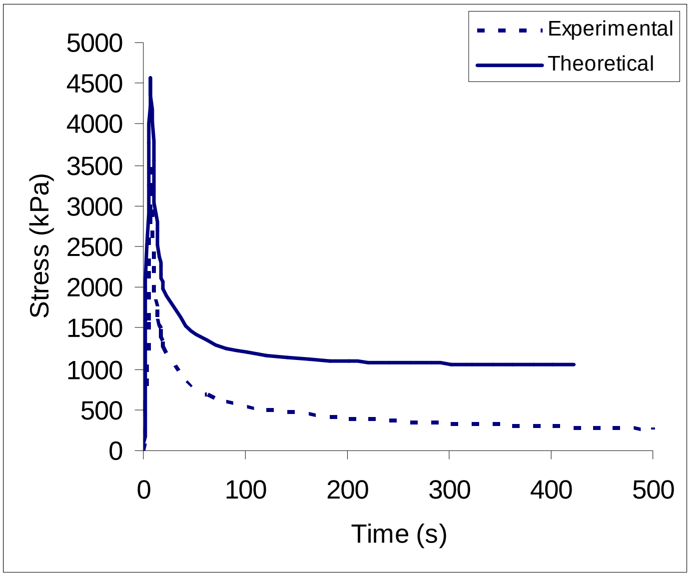

Figure 3: Theoretical predictions of relaxation at 10°C

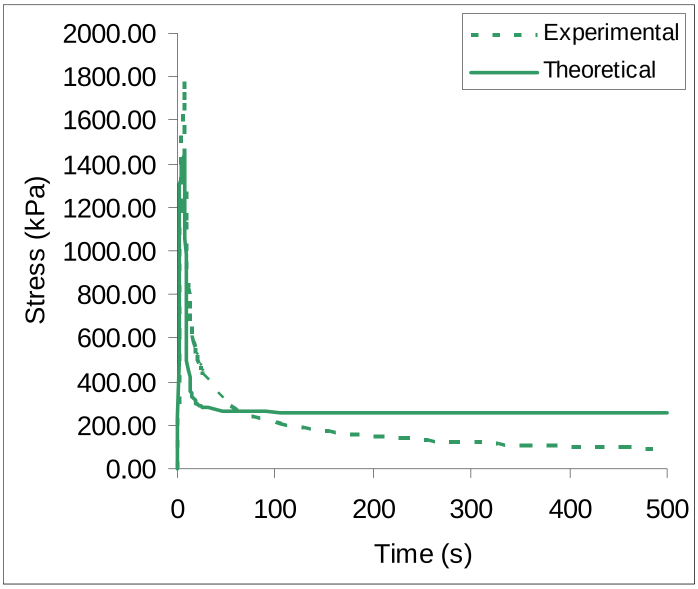

Figure 4: Theoretical predictions of relaxation at 20°C

4.3 Cyclic loading

In the following the theoretical model is used to predict the asphalt response to a trapezoidal load shown in Figure 5. The minimum strain (tension) and maximum strain (compression) are 0.07% and -0.5%. respectively. After reaching the maximum or the minimum strain the specimen is left to rest for 60 seconds. The cyclic load is sustained for 500 seconds. The experimental curves are shown dashed in Figure 6 and Figure 7 for temperatures of 10°C and 20°C respectively. The theoretically predicted curves are shown solid. The theoretical results are in good agreement with the experimental results. The best predictions were obtained for the temperature 20°C. The same pattern of behaviour is predicted as for the experimental analysis.

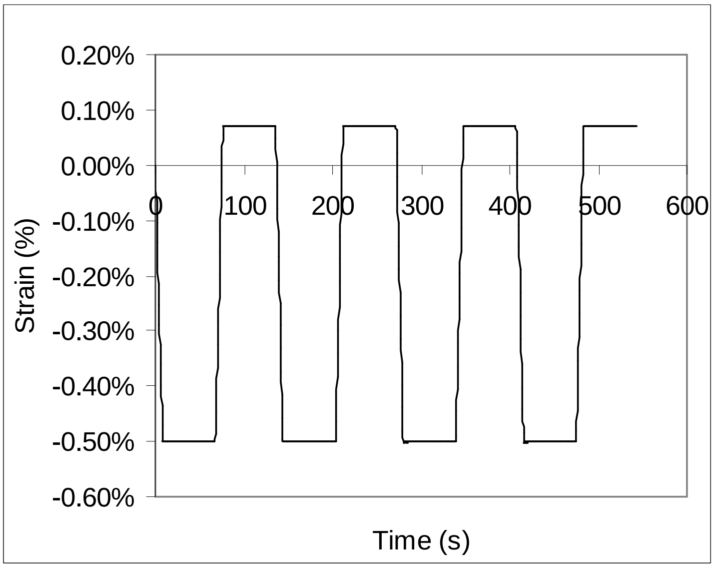

Figure 5: Theoretical predictions of cyclic behaviour at 20°C

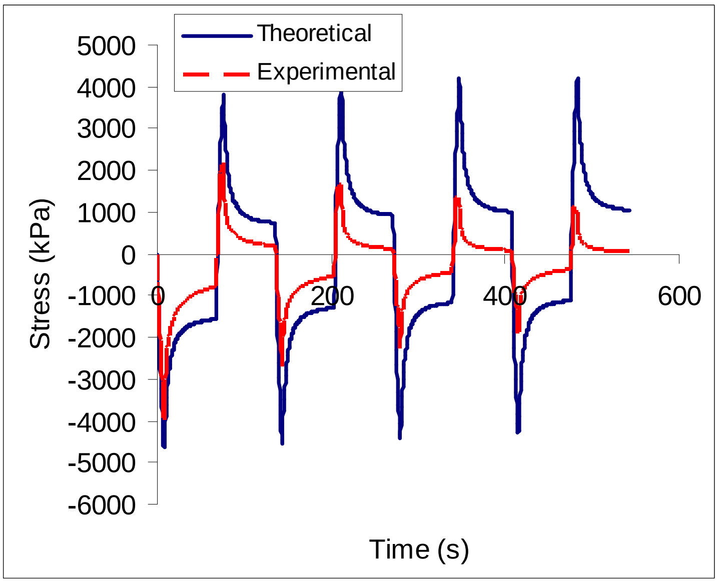

Figure 6: Theoretical predictions of cyclic behaviour at 10°C

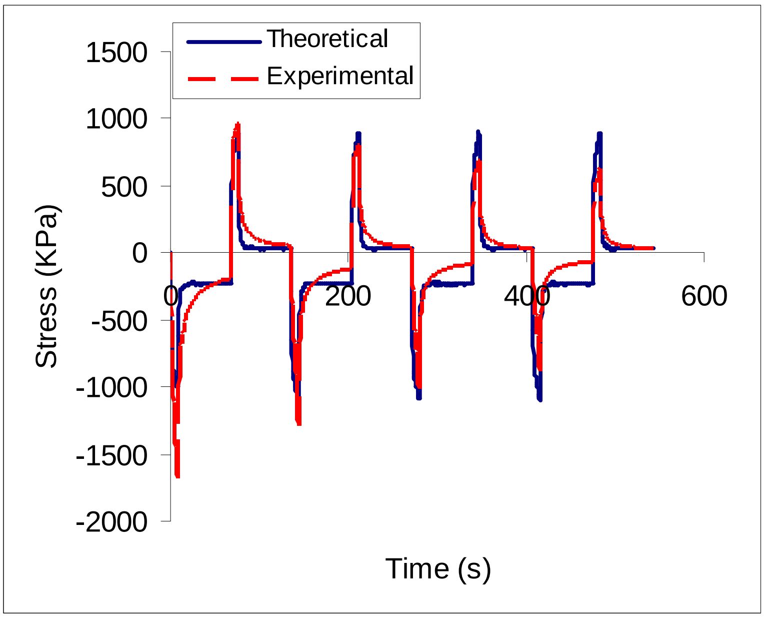

Figure 7: Theoretical predictions of cyclic behaviour at 20°C

5. CONCLUSIONS

A constitutive model based on elasto-viscoplasticity has been developed to predict realistically the load spreading ability of the asphalt material. The behaviour of asphalt is complicated as its mechanical behaviour is very dependent on the rate of loading, the temperature and the material viscosity. The model can accommodate the following shortcomings of the existing elastic model:

- Reduction of stiffness due to viscoplastic flow.
- Increase of stiffness when the rate of loading increases or temperature decrease.
- Occurrence of increased plastic flow at high pavement temperatures.
- Occurrence of viscoplastic relaxation that is dependent on temperature.
- Anisotropy.

The constitutive model was used to predict the behaviour of asphalt in the compression and relaxation tests for different temperatures and different strain rates with a single set of parameter to define the model. Overall the predictions were excellent and this is an important advance on the current elastic models.

REFERENCE

[1] Nesnas, K., *TRF project Task 1*, *Unpublished report*, TRL Limited, Crowthorne, 2002.

[2] Armstrong P.J and Frederick C.O, *General electricity generating board*, *Report No RD/B/N731*, Berkeley Nuclear Laboratories, 1966.

[3] Odqvist, F.K.G., *Mathematical theory of creep and creep rupture*, the Carendon Press, Oxford, 1974.

[4] Lemaitre J and Chaboche J.L., *Mécanique des matériaux solides*, Edn. Dunod, Paris, 1985

[5] Boumahrat M. and Gourdin A., *Methode numeriques appliquées*, Office des publictions universitaires, Algiers, 1983.

[6] Nesnas, K., *TRF project Task 2*, *Unpublished report*, TRL Limited, Crowthorne, 2002.