A New Constitutive model for asphalt

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Summary

The pavement response under a wheel load or temperature variation is assumed to be elastic. The domain of validity of this assumption is restricted only to low temperatures and high speeds where the asphalt layers of the pavement behaves in an elastic manner. At high temperatures the asphalt is subjected to plastic flow associated with viscous behaviour. Clearly the need for a constitutive model to represent the mechanical response of the upper layers of the pavement is necessary. The main components of the model are: elasticity to define the instantaneous response of the model, viscosity to define the dependency of the mechanical response on the strain rate and hence on the frequency of loading, plasticity to define occurrence of the plastic flow and hence permanent deformation. All model parameters such as the Young modulus, the yield stress, the viscosity and the hardening slope in the plastic range are made temperature dependent in order to reflect the sensitivity of asphalt to temperature variations.

Introduction

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Constitutive modelling of asphalt material has lagged behind constitutive modelling of materials such as soils, concrete and metals. The main reason for this is that linear elastic theory has been considered adequate for pavement modelling. The assumption of linear elasticity is correct only at low temperature which produces a high modulus of the asphalt material, and for roads subjected to high-speed traffic.

With slow moving vehicles and hot weather conditions the pavement response is not elastic. As the viscous characteristics of the asphalt are more important and the deformations of the asphalt layers in the pavement are more likely to be controlled by the viscous and the plastic components.

Clearly, to correctly predict the movements in the asphalt layer and the stress distributions requires a constitutive model, which is able to characterise correctly the load spreading ability in pavements. Therefore the constitutive model should account for the following characteristics of the mechanical behaviour of the asphalt material:

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- Temperature: At high temperatures the mechanical behaviour of the asphalt is very dependent on strain rate, due to its viscous nature; at low temperature the behaviour is brittle.
- Frequency of loading: The speed of load application and hence vehicle speed influences the material stiffness. This is very dependent on vehicle speed.
- Anisotropy: It is related to the nature of the material used in pavement construction. Two types of anisotropy can be distinguished: the spatial anisotropy and the mechanical anisotropy. The former relates to material property variations due to material non-homogeneity and the latter relates to change in material strength with plastic straining and the occurrence of different yield stresses for the tension and compression.
- Plastic flow: The occurrence of plastic flow results in stiffness reduction.

The model assumes that viscoplastic strain will not occur until the stress σ is greater than the yield stress σ_y (defining the onset of plasticity). Thus the stress-strain relationship is written as:

$$\sigma = D(\varepsilon - \varepsilon^p) \tag{1}$$

Where σ is the applied stress, D is the tensor of elastic moduli based on Hooke's elasticity, ε is the total strain and ε^p is the viscoplastic strain.

Elasticity

The material elasticity is defined by the following stress-strain relationship relating stress to elastic stain as:

$$\sigma = D\varepsilon^e \tag{2}$$

The components of the tensor of elastic moduli D are defined as:

$$D_{ijkl} = \lambda\delta_{ij} + \mu\lfloor\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\rfloor \tag{3}$$

Where λ is the Lame constant for volume and μ is the Lame constant for shear and δ is the Kronecker constant (δ_{ij}=0 if i=j and δ_{ij}=1 if i≠j). λ and μ are made temperature dependent.

Yield criterion

Kinematic hardening is assumed, it corresponds to a translation of the elastic domain in the stress space during viscoplastic flow. Therefore the stress state is defined by the stress σ and the position of the centre of the yield surface χ. In the following σ and χ are considered to be deviatoric as volumetric deformations in the plastic range are assumed to be negligible.

The onset of viscoplasticity is defined by a simplified version of the general equation of the yield surface[1]. The yield surface is defined as:

$$F = J_2(\sigma - \chi) - \sigma_y - k(\varepsilon_p) = 0 \tag{4}$$

Where $J_2=[2/3(\sigma-\chi):(\sigma-\chi)]^{1/2}$ is the second stress invariant for shear, σ_y is the temperature dependent yield stress, k is the hardening parameter and it is a function of the plastic strain ε^P given as:

$$\dot{k}(\varepsilon^P) = \dot{\chi} = \frac{2}{3} C \dot{\varepsilon}^p - \vartheta \chi \dot{\varepsilon}_p \tag{5}$$

Where the dot symbol indicates first derivative with time; ε_p is the magnitude of the plastic strain ε^P; C and ϑ are coefficients which are dependent on temperature.

Equation (5) was introduced by Armstrong and Frederick[2] for the time-independent plasticity. It is based on an evolution equation of the internal variable χ containing two terms: the first term expresses a linear kinematic hardening and the second term expresses a fading memory effect of the deformation path.

Flow rule

The flow rule is defined based on the Odqvist's law[3] which is a generalization of the well known Norton's law (expressed as a power law of the creep stain related to stress) by considering the elastic domain to be negligible. For this work, a version of the flow rule with an elastic domain developed by Lemaitre et al[4] is used:

$$\dot{\varepsilon}^p = \frac{3}{2}\left[\frac{\left(J_2(\sigma - \chi) - \sigma_y - k(\varepsilon^p)\right)}{\eta}\right]^m \frac{\sigma - \chi}{J_2(\sigma - \chi)} \tag{6}$$

Where $\dot{\varepsilon}^p$ is the viscoplastic strain rate and η and m are constant temperature dependent coefficients.

Parametric identification

Five parameters need to be identified in order to define the constitutive relations, these parameters are: The elastic parameters in terms of the Young Modulus E and the Poisson's ratio ν, the Yield stress σ_y, the viscous characteristics η and the parameter m, and the hardening modulus.

The elastic modulus E is obtained from the initial slope of the stress-strain curves. A value of 0.35 is assumed for Poisson's ratio, generally this value is used for asphalt material. The third parameter which is the yield stress is obtained directly from the relaxation curves. The viscous characteristic η and the parameter m are obtained from fitting Equation (6) to experimental data by using a non-linear least square algorithm[5].

The hardening modulus identifies with C in Equation (6). It is obtained from the relationship linking the variation of the viscoplastic strain with stress level. This relationship is obtained from a plot of stress as a function of viscoplastic strain. The coefficient ϑ is equal to zero.

Theoretical prediction of laboratory tests

In the following, the behaviour of asphalt under compression and relaxation is predicted using the theoretical model. Experimental data[6] showed that the asphalt material is very sensitive to strain rate as observed when plotting the stress-strain curves for asphalt at different strain rates. Also they showed that the asphalt material is very viscous. The effect of strain rate and viscosity increases with an increase of temperature.

The prediction of the material response to compression and relaxation is shown below. The theoretical predictions are show in Figures 1, 2 and 3 for temperature 10^oC, 20^oC and 40^oC. The theoretical results are in good agreement with the experimental results in the pre-peak region. In the post peak region the theoretical model over predicts the experimental results which show a decrease in the stress. This reduction in the stress is because of the damage occurred in the specimen. The modelling of this aspect, which requires coupling of viscoplasticity with damage, has not been attempted yet in the current project. Overall the model prediction are excellent as the right stress level were predicted for all loading rates and temperatures. Most importantly the following observations are made:

- The mechanical response of the asphalt under compression is highly non-linear as it is very dependent on the rate of loading and the temperature of the specimens.
- Failure load increases as the loading rate increases and the temperature decreases.
- The initial slope of the stress-strain curves increases with the increase of the rate of loading and the decrease of the temperature.

As far as the relaxation is concerned, the theoretical results are also in good agreement with the experimental results. The best prediction was for temperature 20^oC and the worst prediction was for 40^oC. The results showed that the asphalt exhibited large stress reduction due to relaxation. All the curves reached an asymptotic value defining the yield stress. The yield stress reduces as the temperature increases. Overall the predictions of the model are excellent as the model parameters obtained from the calibration of the compression tests were used to predict the relaxation tests.

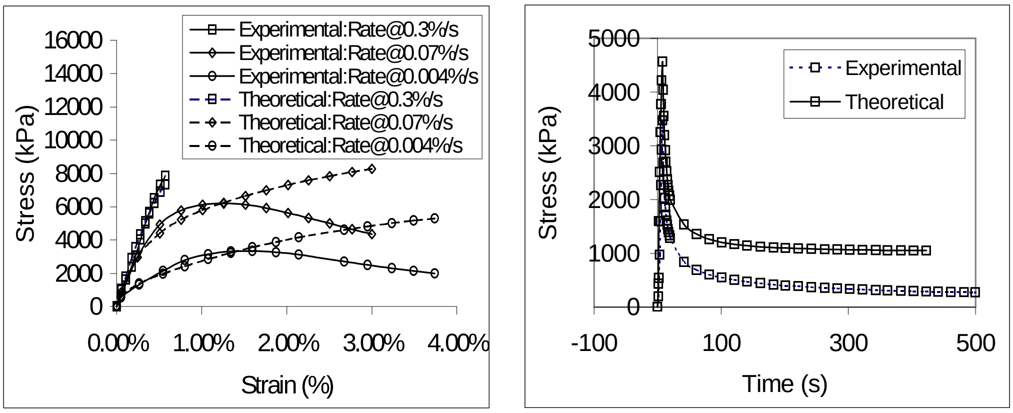

Figure 1. Theoretical predictions of compression and relaxation at 10^{o}C

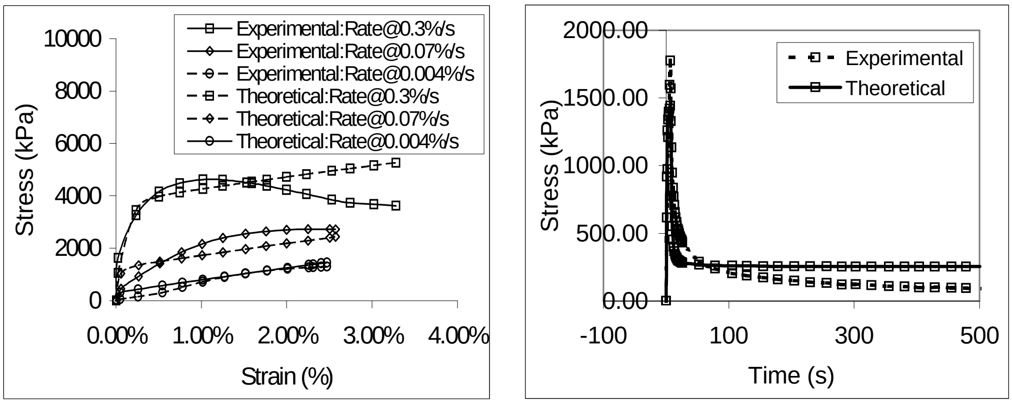

Figure 2. Theoretical predictions of compression and relaxation at 20^{o}C

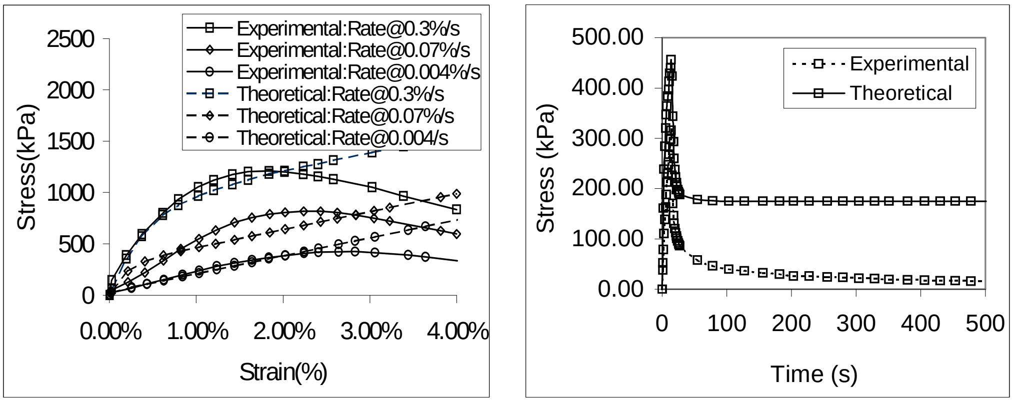

Figure 3. Theoretical predictions of compression and relaxation at 40^{o}C

Conclusions

A constitutive model based on elasto-viscoplasticity has been developed to predict realistically the load spreading ability of the asphalt material. The behaviour of asphalt is complicated as its mechanical behaviour is very dependent on the rate of loading, the temperature and the material viscosity. The model can accommodate the following shortcomings of the existing elastic model:

- Reduction of stiffness due to viscoplastic flow.
- Increase of stiffness when the rate of loading increases or temperature decrease.
- Occurrence of increased plastic flow at high pavement temperatures.
- Occurrence of viscoplastic relaxation that is dependent on temperature.
- Anisotropy.

The constitutive model was used to predict the behaviour of asphalt in the compression and relaxation tests for different temperatures and different strain rates with a single set of parameter to define the model. Overall the predictions were excellent and this is an important advance on the current elastic models.

Reference

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